Tough planar graphs with short longest cycles

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We continue the study of non-Hamiltonian graphs with the property that removing an arbitrary set of vertices disconnects the graph into a relatively small number of components. We present families of maximal planar (chordal planar) such graphs whose longest cycles are short. More formally, the properties which we study are the toughness of graphs and the shortness exponent of classes of graphs.

We improve the upper bound on the shortness exponent of the class of $\frac{5}{4}$-tough maximal planar graphs presented by Harant and Owens (1995). In addition, we present two generalizations of a similar result of Tkáč (1996) who considered 1-tough maximal planar graphs; and we remark that one of these generalizations gives a tight upper bound. We also present two ways of improving the upper bound of Böhme et al. (1999); considering the class of 1-tough planar 3-trees, and 1-tough chordal planar graphs. The main used tools are the 'gluing lemma' of Harant and Owens (which we fix), and the Radoszewski and Rytter's (2011) characterization of trees whose square has a Hamilton path (which we restate using forbidden subgraphs).